# ME2 Computing- Coursework summary

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A) What physics are you trying to model and analyse? (Describe clearly, in words, what physical phenomenon you wish to analyse)

The physics of different types of insects hitting a spider’s web will be analysed with the use of the 1D wave equation. The initial conditions will be defined as the displacement of the web when the insect hits it and how the wave propagates from this point will be evaluated. The 1D wave equation is being used as only a single strand of spider’s web is being investigated. Properties of the spider’s web were found from prior research (Justus, et al., 2022) (Aoyanagi & Okumura, 2010).

B) What PDE are you trying to solve, associated with the Physics described in A? (write the PDE)

This is the 1D wave equation and is a hyperbolic PDE. is the displacement of the web, is the distance along the web, is time and is the wave speed.

C) Boundary value and/or initial values for my specific problem: (be CONSISTENT with what you wrote in A)

The initial displacement will be based on a Gaussian pulse , where maximum displacement is derived from the speed of the insect at impact, using SUVAT. Dirichlet boundary conditions are used where as it is assumed the end of the web will have 0 displacement. Initial velocity of the web is 0. is , (Justus, et al., 2022). Different insects can have varying width, speed and weight.

D) What numerical method are you going to deploy and why? (Describe, in words, which method you intend to apply and why you have chosen it as opposed to other alternatives)

2 numerical methods will be used, 1 to solve the problem explicitly and the other implicitly. The explicit equation used was the central difference method. This method is good because it is simple to implement for the chosen equation and computationally efficient. However, CDM requires the CFL condition to be satisfied for the solution to be stable. This is to make sure that numerical information does not propagate faster than the physical wave speed. The implicit method used was the Crank-Nicolson method. Thus, benefitting from not needing the CFL condition to be met to ensure stability (this will be assessed later). Downsides, however, of implicit methods are the computational costs and the Crank-Nicolson method can introduce oscillations without damping and phase errors in wave propagation.

E) I am going to discretise my PDE as the following (show the steps from continuous to discrete equation and boundary/initial conditions:

Wave Equation (1D):

|  |  |  |
| --- | --- | --- |
|  |  | (1) |

Discretise:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  | (2) |  | (3) |

Substituting (2) and (3) into (1):

|  |  |  |
| --- | --- | --- |
|  |  | (4) |

Implementing Crank-Nicolson method and averaging to the ahead time step for :

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | |  | | | (5) | | |
|  |  | | (6) | Substituting (6) into (5): | | (7) |

Rearranging (7) to form equation (8) for the matrix:

|  |  |  |
| --- | --- | --- |
|  |  | (8) |

This means the diagonalised matrix can be setup as shown in equation (9) (this is just a representation of how the diagonals are filled in, this can be as big as needed in reality):

F) Plot the numerical results comprehensively and discuss them

A graph with different colored lines

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Figure 1: Wave at time steps 0.0🡪2.5s

Figure 1 shows how the wave propagates through the web at different time steps. Time 0.0 shows the initial pulse, with the free oscillations that occur afterwards. It can be seen that the waves energy is conserved but the larger distance steps result in sharp waves.

A group of blue and white graphs

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Figure 2: Web midpoint oscillation with different time steps, thus different CFL conditions

Figure 2 shows the importance of the CFL condition. This is especially critical when using the explicit method to solve the system. When CFL > 1 (bottom left graph with large time step in Figure 2), the solution becomes unstable and cannot be solved. Additionally, when the time step is lowered even with the Crank Nicolson method, the solution becomes much more stable as can be seen in the top row of graphs in Figure 2.

A screen shot of a graph

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Figure 3: 3D plot of wave propagation

Figure 3 shows how the wave propagates with time, it starts as an initial central pulse then this affects the rest of the web until it is all oscillating.

This video shows how the wave propagates through the spider’s web from the initial pulse with CFL 0.00038.

<https://vimeo.com/1068074003/544124e7af?ts=0&share=copy>

|  |  |  |
| --- | --- | --- |
|  |  | (9) |

The first and last rows on the RHS matrix will contain boundary conditions, for example in equation (9).

G) Other remarks (limits of the model, convergence problems, possible alternative approaches, anything you find relevant and important to mention):

The Crank Nicolson method, whilst commonly used for solving parabolic equations like the heat equation, it is not so popular for hyperbolic PDEs like the wave equation. It still works but can introduce oscillations which do not conserve energy and lacks natural damping (Østerby, 2003). Also, the CFL condition affects the stability of the solution, with larger time steps, such as in the top left of Figure 2, causing the wave to dampen and be unstable. A better approach to solving this problem could be using methods such as the implicit Newmark-beta method which is more stable for larger time steps and solves iteratively.

References:

**Østerby, O.** (2003) 'Five ways of reducing the Crank-Nicolson oscillations', *BIT Numerical Mathematics*, 43(4), pp. 811–822. Available at: 10.1023/B:BITN.0000009942.00540.94 (Accessed: 22/03/2025)

**Justus, N., Krugner, R. and Hatton, R.L.** (2022) 'Validation of a novel stereo vibrometry technique for spiderweb signal analysis', *Insects*, 13(4), p. 310. Available at: https://doi.org/10.3390/insects13040310 (Accessed: 13/03/2025).

**Aoyanagi, Y. and Okumura, K.** (2010) 'Simple model for the mechanics of spider webs', *Physical Review Letters*, 104(3), p. 038102. Available at: https://doi.org/10.1103/PhysRevLett.104.038102 (Accessed: 12/03/2025).